

## The Game of NIM: A simple introduction:

The game of NIM is a two person game. The example game arrangement we will play, is 15 objects placed in three columns (Columns go up and down). Column 1 contains 3 objects, Column 2 contains 5 objects, and Column 3 contains 7 objects. NIM can also be played with many other arrangements.

| Column 1 | Column 2 | Column 3 |
| :---: | :---: | :---: |
|  |  |  |
|  |  | x |
|  | x | x |
| x | x | x |
| x | x | x |
| x | x | x |
| $\mathbf{3}$ | x | x |
|  | $\mathbf{5}$ | x |


from "Last Year at Marienbad"

## The Rules:

The players alternate moves.
When it is your move, you may remove as many objects as you wish, but only from a single Column. You cannot 'pass'; you must remove at least one object. The opponent is similarly restricted.

The player who removes the last object is declared the winner (see Variation below).
The previous winner usually politely offers the opponent the choice to go first or second for the next round.

## The Winning Strategy:

There are a couple of winning arrangements that will allow you to often win:

1. Leaving two Column with the same number of objects \& the 3rd column empty.
2. Leaving the three columns with 1, 2 and 3 objects.

However; the game can be played almost perfectly, if you think in the Binary Numbering System, and then follow a few simple steps.

In Binary: (It may help you to think of having Coins of 1, 2 and 4 cent values)

You make 3 with a 1 cent coin and a 2 cent coin.
You make 5 with a 4 cent coin and a 1 cent coin.
You make 7 with a 4 cent coin and a 2 , and a 1 cent coin.
Decimal 3 is 011 in Binary; $(0 \times 4)+(1 \times 2)+(1 \times 1)$
Decimal 5 is 101 in Binary; $(1 \times 4)+(0 \times 2)+(1 \times 1)$
Decimal 7 is 111 in Binary; $(1 \times 4)+(1 \times 2)+(1 \times 1)$


A Few Simple Steps: (Remember - you are thinking in binary)


Using the Binary Values for our starting object arrangement $(3,5,7)$ :

| Binary Value | Column 1 | Column 2 | Column 3 | Count of x's |
| :---: | :---: | :---: | :---: | :---: |
| 4 |  | $x$ | $x$ | 2 |
| 2 | $x$ |  |  | $x$ |
|  | $x$ | $x$ | $x$ | 2 |
|  | $\mathbf{3}$ | $\mathbf{5}$ | $\mathbf{7}$ | 3 |
|  | $\mathbf{1 0 1 1 )}$ | $\mathbf{1 0 1 )}$ | $\mathbf{1 1 1 )}$ |  |
|  |  |  |  |  |
|  |  |  |  |  |

Column 1 has an $x$ for Binary Value 1 and 2 (representing 3).
Column 2 has an x for Binary Value 1 and 4 (representing 5).
Column 3 has an $x$ for Binary Value 1 and 2 and 4 (representing 7).
The "Count of x's" in the last Column holds the key to your success!
Count the x's in each row (rows go across) you will get:


The row for Binary Value 4, The row for Binary Value 2, The row for Binary Value 1 223

In the analysis of NIM it has been determined that there are SAFE object arrangements.
Leaving the board in a SAFE arrangement, assures that you will be the winner; assuming that you continue making SAFE moves for the remainder of the game.

## To be SAFE each row count (Count of $x$ 's) must be even or zero. This is the Key!

It is only possible to move to a SAFE arrangement from an unsafe arrangement.
Similarly; from a SAFE arrangement it is impossible to move to another SAFE arrangement.
The game arrangement we chose is UNSAFE! The $(3,5,7)$ arrangement can be made SAFE by removing 1 object from any column! (changing the "Count of $x$ 's" from 3 to 2 for the Binary Value 1 row)

If you go first, and you remove 1 object from any column. The counts will be:

The row for Binary Value 4, The row for Binary Value 2, The row for Binary Value 1

```
2 2 2
```

All the "Counts of $\mathbf{x}$ 's" are now even, and therefore the game is in a SAFE arrangement. You are the assured Winner!

## just

Some Examples: The examples below are all initially 'unsafe'.

| Binary Value | Column 1 | Column 2 | Column 3 | Count of x's |
| :---: | :---: | :---: | :---: | :---: |
| 4 |  | $x$ | $x$ | 2 |
| 2 |  |  |  | $x$ |
|  | $x$ | $x$ | $x$ | 1 |
|  | 1 | 5 | 7 | 3 |

This example can be made SAFE by removing 3 objects from column 3.
EXAMPLE 2:

| Binary Value | Column 1 | Column 2 | Column 3 | Count of X's |
| :---: | :---: | :---: | :---: | :---: |
| 4 |  |  | x | 1 |
| 2 | x |  | x | 2 |
| 10 | x | x | x | 3 |
|  | 3 | 1 | 7 |  |

This example can be made SAFE by removing 5 objects from column 3.
EXAMPLE 3:

| Binary Value | Column 1 | Column 2 | Column 3 | Count of x's |
| :---: | :---: | :---: | :---: | :---: |
| 4 |  |  |  | $x$ |
| 2 | $x$ |  |  | 1 |
| 1 | $x$ | $x$ | 1 | 4 |

This example can be made SAFE by removing 2 objects from column 3.

I hope that you will think about this (with somebody else or alone) and recognize the beauty of the mathematics that this silly game is based upon.

If you did ... the reward can be life changing!
If you didn't ... there will be many more opportunities!

## Parting Thoughts and Variations:

The rule that you win by taking the last object, is arbitrary (it can be either way).
This rule may be changed during play; but not too late in the game.
There must be 2 non-zero Columns with one Column containing a single object, when the rule must be established. For Example:

| Column 1 | Column 2 | Column 3 |
| :---: | :---: | :---: |
|  |  |  |
| $x$ |  |  |
| $x$ | $x$ |  |



If the rule is 'Last one Wins', you take all but one from the long Column, leaving 2 Columns with 1 object each. They take one and; you win with the last object.
If the rule is 'Last one Loses', you take the entire long Column, leaving 1 Column with 1 object.
They must take the last object and; you win by NOT taking the last object.
This rule change does not influence the early play objective (the SAFE arrangement objective).
What do you do when the arrangement is already SAFE? If your opponent left you with a SAFE arrangement, what then? ...
You encourage your opponent to make a mistake (leaving the board in an UNSAFE arrangement).

1. Make Minimal moves; remove 1 object from the longest Column.
2. Do not remove an entire Column.
3. Talk about a distraction that is interesting and requiring thought.
4. Divert your gaze to an attractive person, place, or thing. Their gaze will follow.
5. Create a loud noise; Drop a Book, Cough, Sneeze, Hum,

Explore the Internet for a vast number of NIM variations; including:


The Number of Columns
The Number of objects in each Column
A Limit on the number of objects removable in one move ...

## For More Exciting Information:

The game of NIM is ...
Proof is HERE ...
Early gaming computer NIMROD ...
Help with BINARY ...

Book of INSTRUCTIONS just \$7,500
An explanation is HERE ...
Early gaming HISTORY ...

